Neural Networks, Dynamical Systems, Control Families, and Formal Languages

蔡永强

北京师范大学

Deep learning has made significant progress in data science and natural science. Some studies have linked deep neural networks to dynamical systems, but the network structure is restricted to residual networks. It is known that residual networks can be regarded as numerical discretizations of dynamical systems. In this talk, we consider traditional network structures and prove that vanilla feedforward networks can also be used for the numerical discretization of dynamical systems, where the width of the network is equal to the input and output dimensions. The proof is based on the properties of the leaky ReLU function and the numerical technique of the splitting method for solving differential equations. The results could provide a new perspective for understanding the approximation properties of feedforward neural networks. In particular, the minimum width of neural networks and the minimal control family of dynamical systems for universal approximation can be derived. In addition, the relationship between mapping compositions and regular languages can be established.

个人简介: 蔡永强, 2017 年取得北京大学计算数学专业理学博士学位, 2017 年 8 月至 2020 年 8 月在新加坡国立大学理学院做博士后研究工作。2020 年 9 月进入北京师范大学数学科学学院工作。研究兴趣包括高分子自组装、液晶相双层膜的数值模拟，以及深度学习的理论、算法和应用。研究成果发表在 Advanced Functional Materials, Soft Matter, Journal of Chemical Physics 等学术期刊以及机器学习国际顶级学术会议 ICML, ICLR 上。

Linking causal and structural connectivity in pulse-output nonlinear networks

陈开

上海交通大学

The causal connectivity of a network is often inferred to understand network function. It is arguably acknowledged that the inferred causal connectivity relies on the causality measure one applies, and it may differ from the network's underlying structural connectivity. However, the interpretation of causal connectivity remains to be fully clarified, in particular, how causal connectivity depends on causality measures and how causal connectivity relates to structural connectivity. Here, we focus on nonlinear networks with pulse signals as measured output, e.g., neural networks with spike output, and address the above issues based on four commonly utilized causality measures, i.e., time-delayed correlation coefficient, time-delayed mutual information, Granger causality, and transfer entropy. We theoretically show how these causality measures are related to one another when applied to pulse signals. Taking a simulated Hodgkin–Huxley network and a real mouse brain network as two illustrative examples, we further verify the quantitative relations among the four causality measures and demonstrate that the causal connectivity inferred by any of the four well coincides with the underlying network structural connectivity, therefore illustrating a direct link between the causal and structural connectivity. We stress that the structural connectivity of pulse-output networks can be reconstructed pairwise without conditioning on the global information of all other nodes in a network, thus circumventing the curse of dimensionality. Our framework provides a practical and effective approach for pulse-output network reconstruction.

个人简介：我本科毕业于上海交通大学物理与天文学院，目前在本校数学科学学院攻读应用数学博士学位。我的研究领域是计算神经科学，主要通过数学物理建模和数值计算来研究大脑的结构与功能，最终希望理解认知与意识的本质。我的研究兴趣包括运用统计推断方法分析大脑结构，以及设计基于大脑网络结构约束的循环神经网络，以探究大脑执行高级认知功能的计算机制。

Bound-Preserving OEDG Schemes for Aw-Rascle-Zhang Traffic Models on Networks

陈威

厦门大学

Physical solutions to the widely used Aw–Rascle–Zhang (ARZ) traffic model and the adapted pressure (AP) ARZ model should satisfy the positivity of density, the minimum and maximum principles with respect to the velocity v and other Riemann invariants. Many numerical schemes suffer from instabilities caused by violating these bounds, and the only existing bound-preserving (BP) numerical scheme (for ARZ model) is random, only first-order accurate, and not strictly conservative. This paper introduces arbitrarily high-order provably BP discontinuous Galerkin (DG) schemes for these two models, preserving all the aforementioned bounds except the maximum principle of v, which has been rigorously proven to conflict with the consistency and conservation of numerical schemes. Although the maximum principle of v is not directly enforced, we find that the strictly preserved maximum principle of another Riemann invariant w actually enforces an alternative upper bound on v. At the core of this work, analyzing and rigorously proving the BP property is a particularly nontrivial task: the Lax–Friedrichs (LF) splitting property, usually expected for hyperbolic conservation laws and employed to construct BP schemes, does not hold for these two models. To overcome this challenge, we formulate a generalized version of the LF splitting property, and prove it via the geometric quasilinearization (GQL) approach. To suppress spurious oscillations in the DG solutions, we incorporate the oscillation-eliminating (OE) technique, which is based on the solution operator of a novel damping equation. Several numerical examples are included to demonstrate the effectiveness, accuracy, and BP properties of our schemes, with applications to traffic simulations on road networks.

个人简介：陈威，厦门大学博士二年级在读，2021 年本科毕业于福州大学，并于同年进入厦门大学攻读硕士研究生，2023 年获硕博连读资格。目前，陈威的主要研究方向为磁流体方程的渐进保持格式及交通流方程的保界方法设计。至今已在《Journal of Computational Physics》发表两篇论文，并在《Journal of Scientific Computing》发表一篇论文。2024 年 8 月，陈威获得国家公派出国留学资格。

基于常微分方程的采样算法与快速生成模型

丁钊

武汉大学

我将从插值的角度引入桥接两个概率分布的常微分方程系统,并针对经典采样问题和数据生成问题,设计对应的高效计算格式,并给出适定性和收敛性分析。

针对采样问题，人们发现，即使对于简单的多峰分布，主流 MCMC 方法的混合时间可能趋于无穷。我们的方法从原理上可以可以克服这一困难，通过具体数值算例，我们证实对于复杂的多峰分布，我们的方法具有更好的精度。

针对生成问题，我们论证提出的格式与扩散模型的相合性，而主流扩散模型受限于其模型原理，采样速度天然慢于其他生成模型。我们利用常微分方程轨迹的确定性，通过蒸馏机制，设计稳定的神经网络参数化方法，得到从噪声直接一步生成数据的扩散模型，并在经典图片数据集上通过指标说明算法的优越性。

个人简介: 丁钊。2020 年本科毕业于武汉大学数学与统计学院数学基地班，同年在该学院攻读计算数学博士，师从杨志坚教授。研究方向包括深度学习理论与扩散模型理论和应用。早期经历包括: 高性能数学函数库设计和实现、过参数化场景下 PINNs 的收敛性分析等，近期主要关注扩散模型等加速算法和扩散模型的交叉应用。我的研究兴趣是具有现实应用场景和意义的数学问题。现有两篇工作接收于《Communications in Computational Physics》。

二维柱坐标系下求解流体力学方程的二阶保球对称守恒拉格朗日间断有限元格式

冯文静

北京应用物理与计算数学研究所

拉格朗日方法是模拟多介质流体流动的主要数值方法之一,其特点是网格跟随流体移动，因此在移动单元边界上没有质量交换。本次报告针对可压缩欧拉方程提出了一种真正二阶精度单元中心型拉格朗日间断有限元（DG）格式。该拉氏 DG 格式是基于物理坐标系下的四边形网格，直接求解守恒变量，因此可以自然保持质量、动量和总能量的守恒性质。进一步地，基于三维柱对称内爆问题的研究需求，在二维柱坐标系下设计了一种同时保球对称和守恒性质的二阶拉氏 DG 格式。该格式实际上是体格式和面格式的结合，从理论上可以证明在初始等角射线网格上计算时能够保持一维球对称性质。为了控制数值振荡，我们采用多分辨率 WENO 限制器。该限制器不会破坏守恒、精度和对称性质。数值实验表明以上格式具有二阶精度、对称和不振荡的优良性能。

个人简介：北京应用物理与计算数学研究所 2022 级博士生，导师为成娟研究员，研究方向是计算流体力学。主要研究内容是求解流体力学方程的高精度守恒拉格朗日间断有限元方法。在 Journal of Computational Physics 和 Advances in Water Resources 期刊上发表论文 3 篇。

Probabilistic error analysis of CholeskyQR based on columns

关浩然

香港理工大学

CholeskyQR-type algorithms are very popular in both academia and industry in recent years. It could make a balance between the computational cost, accuracy and speed. CholeskyQR2 provides numerical stability of orthogonality and Shifted CholeskyQR3 deals with problems regarding ill-conditioned matrices. 3C algorithm is applicable for sparse matrices. However, the overestimation of the error matrices in the previous works influences the sufficient conditions for these algorithms. Particularly, it leads to a conservative shifted item in Shifted CholeskyQR3 and 3C, which may greatly influence the properties of the algorithms. In this work, we consider the randomized methods and utilize the model of probabilistic error analysis of Nick Higham to do rounding error analysis for CholeskyQR-type algorithms. We combine the theoretical analysis with the g-norm defined in the previous work. Our analysis could provide a smaller shifted item for Shifted CholeskyQR3 and could improve the orthogonality of our 3C algorithm for dense matrices. Numerical experiments in the final section shows that our improvements with randomized methods do have some advantages compared with the original algorithms.

个人简介：关浩然，香港理工大学应用数学系博士生，导师为乔中华教授及樊玉伟博士。2020 年本科毕业于大连理工大学。主要研究方向为数值线性代数及矩阵分析，尤其是矩阵的范数估计，舍入误差分析，特征值与奇异值性质及随机数值代数方法等。曾在意大利博洛尼亚大学数学系进行学术交流及课题研究。曾在中国工业与应用数学学会年会，SIAM ALA 2024 等会议上做学术报告。

A positivity-preserving finite difference scheme for the Flory-Huggins-Cahn-Hilliard equation with dynamical boundary condition

郭云卓

北京师范大学

We proposed and analyzed a finite difference numerical scheme for the Flory-Huggins-Cahn-Hilliard equation with dynamical boundary condition. The dynamical evolution equation for the boundary profile corresponds to a lower-dimensional logarithmic energy potential. In turn, a theoretical analysis for the coupled system becomes very challenging. In the numerical design, a convex splitting approach is applied to the chemical potential associated with the energy in both bulk and boundary level. Finite difference approximation and convexity analysis reveal that such a numerical system could be represented as a minimization of a discrete convex functional. The singularity fact prevents the solution from approaching the singular points and ensures the unique solvability. The total energy stability analysis could be fulfilled by an estimate over the finite difference inner product. In addition, an H^{-1} convergence analysis is established via a carefully designed correction function. Some numerical results are presented, which demonstrate the robustness of the numerical scheme.

个人简介：报告人郭云卓目前就读于北京师范大学数学科学学院，师从张争茹教授开展研究工作。主要研究具有动力边界条件的梯度流问题，保正或保结构数值格式和理论分析，以及非线性方程组的快速算法。文章在 Mathematics of Computation 和 Journal of Scientific Computing 杂志上发表。

A phase field model for deformation-induced amorphization

黄运通

香港科技大学

Amorphization due to severe plastic deformation has been discovered in various crystalline materials. Despite its importance, developing a rigorous and general theory of strain-induced amorphization remains elusive due to the intricate nature of modelling microstructural changes and deformation mechanisms. In this study, we propose a novel model integrated with elastic-plastic theory to shed light on shear-induced amorphization in nanocrystalline alloys. Our model incorporates the martensitic transformation of the austenite phase under large plastic deformation, followed by the intensification of crystal fracture on the martensite phase to form an amorphous phase. Simulations suggest that amorphous nucleation is more likely to occur in high-stress regions, such as shear bands, and that the critical plastic strain for amorphization increases as grain size enlarges. These observations align with experimental data, indicating that our phase-field model captures the physical picture of shear-induced amorphization and can predict the threshold for amorphization. Overall, our work offers valuable insights into shear-induced amorphization and paves the way for enhancing the understanding of amorphous materials and fostering the development of more precise and comprehensive models for investigation.

个人简介：作为 2023 级香港科技大学在读博士研究生，我对材料建模与计算充满热情与探索精神。受香港博士研究生奖学金资助计划（HKPFS）的支持，成功在形变诱导非晶化研究方面取得了进展。本科毕业于中南大学，并获得 2018 年国家奖学金，后来在武汉大学继续深造，取得硕士学位。曾在第十六届世界计算力学大会（WCCM 2024）上分享非晶化相场模型的研究成果。

Natural model reduction for kinetic equations

金则宇

北京大学

Numerical simulation of kinetic equations poses significant challenges due to their inherently high-dimensional nature. This talk introduces a novel geometric approach to achieve model reduction while preserving essential structural properties of the equations under certain conditions. By employing projections onto tangent bundles of finite-dimensional approximate solution manifolds, our framework naturally yields first-order hyperbolic systems. We introduce criteria for selecting Riemannian metrics for kinetic equations, which act as analogues of symmetrizers for first-order PDEs, ensuring hyperbolicity and linear stability. Furthermore, we establish, for the first time, a rigorous connection between the H-theorem for kinetic equations and the linear stability conditions for the reduced models. Joint work with Ruo Li.

个人简介：金则宇是北京大学数学科学学院计算数学专业 2021 级博士研究生，师从李若教授。他于 2021 年获得北京大学学士学位，其研究方向为微分方程数值解。

Feature Averaging: An Implicit Bias of Gradient Descent Leading to Non-Robustness in Neural Networks

李柄辉

北京大学

In this work, we investigate a particular implicit bias in the gradient descent training process, which we term “Feature Averaging”, and argue that it is one of the principal factors contributing to non-robustness of deep neural networks. Despite the existence of multiple discriminative features capable of classifying data, neural networks trained by gradient descent exhibit a tendency to learn the average (or certain combination) of these features, rather than distinguishing and leveraging each feature individually. In particular, we provide a detailed theoretical analysis of the training dynamics of gradient descent in a two-layer ReLU network for a binary classification task, where the data distribution consists of multiple clusters with orthogonal cluster center vectors. We rigorously prove that gradient descent converges to the regime of feature averaging, wherein the weights associated with each hidden-layer neuron represent an average of the cluster centers (each center corresponding to a distinct feature). It leads the network classifier to be non-robust due to an attack that aligns with the negative direction of the averaged features. Furthermore, we prove that, with the provision of more granular supervised information, a two-layer multi-class neural network is capable of learning individual features, which is able to induce a binary classifier with the optimal robustness under our setting. Besides, we also conduct extensive experiments using synthetic datasets, MNIST and CIFAR-10 to substantiate the phenomenon of feature averaging and its role in adversarial robustness of neural networks. We hope the theoretical and empirical insights can provide a deeper understanding of the impact of the gradient descent training on feature learning process, which in turn influences the robustness of the network, and how more detailed supervision may enhance model robustness.

个人简介：李柄辉，北京大学前沿交叉学科研究院国际机器学习研究中心 2023 级博士生，本科毕业于北京大学信息科学技术学院图灵班，曾荣获北京大学校长奖学金、北京大学校级三好学生以及北京大学“未名学士”荣誉称号等奖励荣誉。其主要研究兴趣为深度学习理论（特别在对抗鲁棒性问题、特征学习理论以及大模型的缩放定律等方向）与机器学习在数学中的应用（特别是利用机器学习算法求解高维组合数学问题）。

Function and Derivative Approximation by Shallow Neural Networks

李圆媛

复旦大学

We investigate a Tikhonov regularization scheme specifically tailored for shallow neural networks within the context of solving a classic problem: approximating an unknown function and its derivatives in a unit cubic domain based on noisy measurements. The proposed Tikhonov regularization scheme incorporates a penalty term that takes three distinct yet intricately related network (semi)norms: the extended Barron norm, the variation norm, and the Radon-BV seminorm. These choices of the penalty term are contingent upon the specific architecture of the neural network being utilized. We establish the connection between various network norms and particularly trace the dependence of the dimensionality index, aiming to deepen our understanding of how these norms interplay with each other. We revisit the universality of function approximation through various norms, establish rigorous error-bound analysis for the Tikhonov regularization scheme, and explicitly elucidate the dependency of the dimensionality index, providing a clearer understanding of how the dimensionality affects the approximation performance and how one designs a neural network with diverse approximating tasks.

个人简介：李圆媛，复旦大学数学科学学院计算数学博士研究生，师从陆帅教授，研究方向为神经网络的数学理论，主要探索相关空间和逼近理论的应用。曾获国家留学基金委奖学金，并赴维也纳大学访问一年。在《Numerische Mathematik》期刊发表论文，多次参与国内外重要学术会议并作报告。李圆媛的研究目标是通过深入的数学分析，推动神经网络的发展，为人工智能领域提供坚实的理论基础。

From Generalization Analysis to Optimization Designs for State Space Models

刘馥盛

新加坡国立大学

In this talk, I will discuss the role of State Space Models (SSMs) in time series analysis and their potential as an alternative to transformers for sequence modeling. I will introduce the generalization theory of SSMs and propose improvements to how these models are trained. By examining the connection between SSM parameters and the temporal patterns in training data, we can: (1) develop a scaling rule for model initialization that improves the consistency of SSM outputs across various data patterns, and (2) introduce a new training regularization method to enhance model performance. These insights offer practical benefits for using SSMs effectively in sequence modeling tasks.

个人简介:我叫刘馥盛,目前是新加坡国立大学数据科学系的五年级博士生,本科毕业于西安交通大学数学试验班,现师从李千骁老师。我的研究兴趣集中在深度学习的优化和泛化理论,以及它们之间的相互联系。我特别关注状态空间模型在序列建模中的理解,及其在语言模型中的应用,比如提升训练效率和推理能力等课题。

Low-rank optimization on Tucker tensor varieties

彭任锋

中国科学院数学与系统科学研究院

In the realm of tensor optimization, the low-rank Tucker decomposition is crucial for reducing the number of parameters and saving storage. In this talk, we delve into the geometry and optimization methods for Tucker tensor varieties---the set of tensors with bounded Tucker rank---which is notably more intricate than the well-explored matrix varieties. We give an explicit parametrization of the tangent cone of Tucker tensor varieties and leverage its geometry to develop provable gradient-related line-search methods for optimization on Tucker tensor varieties. In practice, low-rank tensor optimization suffers from the difficulty of choosing a reliable rank parameter. To this end, we incorporate the established geometry and propose a Tucker rank-adaptive method that aims to identify an appropriate rank. Numerical experiments on tensor completion reveal that the proposed methods are in favor of recovering performance over other state-of-the-art methods. This is joint work with Bin Gao (AMSS) and Ya-xiang Yuan (AMSS).

个人简介：彭任锋，中国科学院数学与系统科学研究院 2021 级直博生，导师袁亚湘研究员，本科毕业于同济大学数学科学学院。曾任中国科学院—SIAM 学生分会主席，获得过中国科学院华罗庚奖学金等荣誉。他的研究兴趣集中在带低秩约束的矩阵和张量优化问题，特别是利用约束集的几何结构设计优化方法，相关成果发表在 computational optimization and applications 等期刊。

Interface Laplace Learning: Learnable Interface Term Helps Semi-Supervised Learning

王汤军

清华大学

We introduce a novel framework, called Interface Laplace learning, for graph-based semi-supervised learning. Motivated by the observation that an interface should exist between different classes where the function value is non-smooth, we introduce a Laplace learning model that incorporates an interface term. This model challenges the long-standing assumption that functions are smooth at all unlabeled points. In the proposed approach, we add an interface term to the Laplace learning model at the interface positions. We provide a practical algorithm to approximate the interface positions using k-hop neighborhood indices, and to learn the interface term from labeled data without artificial design. Our method is efficient and effective, and we present extensive experiments demonstrating that Interface Laplace learning achieves better performance than other recent semi-supervised learning approaches at extremely low label rates on the MNIST, FashionMNIST, and CIFAR-10 datasets.

个人简介：王汤军，于 2020 年在清华大学数学系获得理学学士学位，目前在清华大学丘成桐数学中心史作强教授指导下攻读博士学位。研究领域包括半监督学习，微分方程在深度学习中的应用，非局部模型等。工作发表于 TPAMI，CMS 等期刊。

Discontinuous Galerkin methods for the steady-state solutions of Euler equations

韦磊

中国科学技术大学

In the realm of steady-state solutions of Euler equations, the pursuit of residue convergence to machine precision has been a persistent challenge for high-order shock-capturing schemes, especially in the presence of intense shock waves. To address this challenge, we have introduced a hybrid limiter within the framework of discontinuous Galerkin (DG) methods. This limiter integrates the jump indicator and limiter components seamlessly, yielding a more cohesive and efficient approach. For steady-state problems, we have utilized the hybridization of the DG solution with the cell average, eliminating the necessity for characteristic decomposition and intercell communication, thereby significantly reducing computational costs and enhancing parallel efficiency. Additionally, we have developed a novel jump filter, which operates locally based on jump information at cell interfaces, targeting high-order polynomial modes within each cell. This filter not only retains the localized nature but also preserves the key attributes of the DG method, including conservation, L2 stability, and high-order accuracy. We have also explored its compatibility with other damping techniques and demonstrated its seamless integration into a hybrid limiter. Numerical experiments are presented to illustrate the robust performance of these schemes for steady Euler equations on both structured and unstructured meshes.

个人简介：韦磊是中国科学技术大学数学科学学院在读博士生，导师是中国科学技术大学夏银华教授和徐岩教授。主要研究方向是高精度数值方法，包括间断有限元和加权本质无震荡格式的研究，应用于计算流体、天体物理等方面的数值模拟。相关工作发表在 Journal of Computational Physics 期刊杂志。

Phase field based thermal--fluid topology optimization method

夏青

西安交通大学

In this talk, we propose a novel TPMS based topology optimization method for the hydrodynamical and convective heat transfer, which can optimize the channels shapes to maximize the heat loss and mass flux by holding the constant mean curvature. The fluids channels with non-trivial geometries freely evolve along the optimization process with the TPMS-based assumption. We proposed a new energy functional by modifying the original energy equation based on the constant curvature property of the TPMS structure, which leads to a novel governing system composed of the Allen–Cahn type model, the Darcy–Stokes model, and the convection–diffusion heat transfer model. The Crank–Nicolson method with second order time accuracy and the central difference method with second order space accuracy were utilized to discretize the system. The Lagrange multiplier method was applied to avoid the influence of higher-order nonlinear terms and highly coupled terms on the stability of the proposed scheme. It was proved that the system satisfied the unconditional energy dissipation of the original energy in both continuous and discrete cases, which indicated that large-scale computation and large time steps can be performed. We performed various numerical experiments and carried out topology optimization based on non-uniform TPMS structure with different porosity, types, and sizes, which demonstrated that the fluid channel with optimized structures can greatly improve heat transfer than the original TPMS-based channel from the quantitative and qualitative perspectives.

个人简介：夏青，西安交通大学数学与统计学院在读博士生，师从李义宝教授。主要从事多物理场耦合建模与计算、3D 打印的图形处理与拓扑优化等方面的研究。个人获得过“2024 年度陕西高等学校科学技术研究优秀成果一等奖（第三完成人）”，西安交通大学“2022 年度优秀研究生标兵（个人最高奖）”等多项荣誉，多次荣获“国家奖学金”。在 CMAME,JCP,PF 等国内外重要学术刊物发表论文二十余篇（其中一篇 Feature 文章，并接受 AIP 专访；三篇 ESI 高被引）。

Weak Galerkin finite element method for interface problems with curved interface

杨琳

吉林大学

In this work, we use the weak Galerkin (WG) finite element method to solve interface problems with curved interface. When solving such problems on fitted meshes, the geometric error introduced by approximating the curved interface with straight segments limits the accuracy of high-order numerical methods. To overcome this challenge, we directly construct the WG space on curved interface elements, thus avoiding geometric error. To demonstrate the effectiveness of this method, we apply it to the Stokes interface problem as an example. However, this method may become inefficient for problems with moving interfaces, as the mesh has to be updated to capture the evolving interface. Therefore, we consider solving interface problems on unfitted meshes. In our method, standard finite element spaces are used in non-interface elements, while immersed weak function spaces that exactly satisfy interface conditions are employed in the interface elements. The immersed interface function space is constructed to maintain optimal approximation properties. At the same time, the proposed numerical scheme achieves optimal convergence rates. We demonstrate the effectiveness of the scheme using a second-order elliptic interface problem as an example.

个人简介：杨琳，吉林大学数学学院计算数学博士研究生。目前主要研究方向为非标准有限元方法，包括贴体网格上的曲边交界面问题、非贴体网格上浸入界面函数的构造和应用等。相关研究成果发表在 Journal of Computational Physics、Applied Numerical Mathematics 等期刊上。

A fast numerical scheme for fractional viscoelastic models of wave propagation

原豪

四川大学

We propose a fast scheme for approximating the Mittag-Leffler function by an efficient sum-of-exponentials (SOE), and apply the scheme to the viscoelastic model of wave propagation with mixed finite element methods for the spatial discretization and the Newmark-beta scheme for the second-order temporal derivative. Compared with traditional L1 scheme for fractional derivative, our fast scheme reduces the memory complexity from $\mathcal O(N_sN)$ to $\mathcal O(N_sN_{exp})$ and the computation complexity from $\mathcal O(N_sN^2)$ to $\mathcal O(N_sN_{exp}N)$, where N denotes the total number of temporal grid points, N_{exp} is the number of exponentials in SOE, and N_s represents the complexity of memory and computation related to the spatial discretization. Numerical experiments are provided to verify the theoretical results.

个人简介：原豪，四川大学数学学院博士研究生，导师为谢小平教授。主要研究领域为整数阶和时间分数阶粘弹性波传播模型的数值方法。作为主要完成人，参与了中国核动力研究设计院《基于有限元的 Pn 中子输运计算方法研究实现》项目和华为技术有限公司《LAPACK_压缩格式线性系统求解函数》项目。获得了 2022 年“鲲鹏众智金质量奖”和四川大学 2022-2023 学年优秀研究生奖。

Selective focusing of multiple particles in homogeneous and layered medium

张金锐

浙江大学

Inverse scattering of multiple well-separated particles has a wide range of applications, but remains highly challenging due to the multiple scattering effect. In this talk, we apply the DORT (Decomposition of the Time Reversal Operator) method to this imaging problem, enabling selective focusing on each particle using far field measurements. Through asymptotic analysis of the time reversal operator and the decay properties of oscillatory integrals, we establish a relationship between the eigenfunctions of the time reversal operator and the locations of the particles in homogeneous and layered medium. Specifically, we show that under suitable conditions, each particle with various boundary generates several significant eigenvalues with the corresponding eigenfunctions producing incident waves that focus selectively on the associated particle. Application of this method to imaging particles with multiple scattering effect is demonstrated through boundary integral formulations.

个人简介：张金锐，南京理工大学数学系本科，浙江大学数学科学学院博士生，导师为赖俊教授。主要研究声波及弹性波方程的散射与反散射问题，在数学知名杂志 Inverse Problems、SIAM Journal on Numerical Analysis 发表文章。

Stability of Least Square Approximation under Random Sampling

张鑫悦

中国科学院数学与系统科学研究院

This paper investigates the stability of the least squares approximation P_m^n within the univariate polynomial space of degree m, denoted by P_m. The approximation P_m^n entails identifying a polynomial in P_m that approximates a function f over a domain X based on samples of f taken at n randomly selected points, according to a specified measure. The primary goal is to determine the sampling rate necessary to ensure the stability of P_m^n. Assuming the sampling points are i.i.d. with respect to a Jacobi weight function, we present the sampling rates that guarantee the stability of P_m^n. Specifically, for uniform random sampling, we demonstrate that a sampling rate of n \asymp m^2 is required to maintain stability. By integrating these findings with those of Cohen-Davenport-Leviatan, we conclude that, for uniform random sampling, the optimal sampling rate for guaranteeing the stability of P_m^n is n \asymp m^2, up to a log(n) factor. Motivated by this result, we extend the impossibility theorem, previously applicable to equally spaced samples, to the case of random samples, illustrating the balance between accuracy and stability in recovering analytic functions.

个人简介：张鑫悦，中科院数学与系统科学研究院计算数学与科学工程计算研究所 2024 级博士研究生，师从许志强教授。我的主要兴趣是数据科学和数值逼近，研究从数据中恢复对象的理论。我目前的研究聚焦于数值逼近领域的理论和算法，关心数值逼近方法（如插值、最小二乘等）的基本问题，即收敛性、稳定性、计算效率和样本复杂度，特别关注采样问题。

A local deep learning method for solving high order partial differential equations

朱全会

南方科技大学

Deep learning-based methods are being employed to resolve the computational challenges of high-dimensional partial differential equations (PDEs). But the computation of the high order derivatives of neural networks is costly, and high order derivatives lack robustness for training purposes. This work proposes an approach to solve PDEs with high order derivatives. Numerical examples are carried out to demonstrate that our local deep learning is efficient, robust, flexible, and is particularly well-suited for high-dimensional PDEs with high order derivatives. Moreover, a priori error estimate of the method is derived, which is the first theoretical study of this method. It is proved that the neural network solutions will converge if the training samples and network size are large enough without any constraint on the ratio of training samples to the network size. Besides, our results suggest that the method can recover high order derivatives better than the deep Ritz method, which has also been verified by numerical experiments.

个人简介：朱全会，南方科技大学数学系博士研究生，计算数学方向，导师为杨将老师。本科毕业于浙江大学数学系信息与计算科学专业。研究兴趣为深度学习与科学计算的结合，关心神经网络运行中的普遍问题，及其在计算数学中的应用。主要研究成果发表在 J Sci. Comput.，高等学校计算数学学报(英文版)中。